Peak to peak amplitudes of 10 fps were obtained without structural reinforcement. Phase measurements showed that plane waves were moving upstream at near sound velocities. Wave forms were closely sinusoidal (see Fig. 3).

References

[1] Miller, J. A. and Fejer, A. A., "Transition Phenomena in Oscillating Boundary Layer Flows," *Journal of Fluid Mechanics*, Vol. 18, 1964, p. 438.

[2] Thomas, R. E. and Morland, B. T., Jr., "Gust Simulation in a Wind Tunnel," Space Technology Rept. 67-52, July 1967, Texas A&M Univ., College Station, Texas.

[3] Donovan, A. F. and Lawrence, H. R., ed., "Aerodynamic Components of Aircraft at High Speeds," *High Speed Aerodynamics and Jet Propulsion*, Vol. VII, Sec. F, Princeton University Press, 1957, pp. 671–675.

A Direct Matrix Method for the Divergence Problem

V. T. Nagaraj*
Hindustan Aeronautics Ltd., Bangalore, India

Nomenclature

a = flexibility matrix
A = see Eq. (9)
B = see Eq. (9)
c = wing chord
C_h = aerodynamic influence coefficient matrix
$C_{L\alpha}$ = lift curve slope
$C_{m\alpha}$ = moment curve slope
C^z = translation flexibility (two-dimensional model)
C^α = rotational flexibility (two-dimensional model)
d = see Fig. 2
e = see Fig. 2
F = column matrix of control point forces
h = column matrix of the control point deflections
I = unit matrix
S = area of two-dimensional wing model
q = dynamic pressure

Introduction

In the estimation of the divergence speed of a lifting surface, the usual procedure is to compute the moment, about a convenient reference axis, due to the aerodynamic forces and to equate this to the structural restoring moment. When the surface has a well defined elastic axis, this procedure is simple to apply and there are a number of methods for obtaining the divergence speed, if the bending and the torsional stiffnesses are known.[1] However, in many cases it may not be possible to separate the two effects and the stiffnesses are usually available in the form of a matrix of structural influence coefficients measured at a set of points on the surface. In the following, a method is given by which a direct solution can be obtained for the divergence speed using the matrices of the structural and the static aerodynamic influence coefficients.

Table 1 Symmetrical divergence of the wing of Fig. 3

Method	Divergence speed (fps)	Comments
Ref. 1	1948.2	Strip theory
Ref. 1	1823.1	Lifting line theory
Present	1583.2	One station at 268 in.
Present	1704.8	Two stations (186 in. and 368 in.)
Present	1919.2	Three stations (90 in., 268 in. and 468 in.)
Present	1925.1	Five stations (Fig. 3)
Present	1786.5	Five stations; lifting line theory

Received June 2, 1971.

* Project Engineer (Aeroelasticity). Member AIAA.

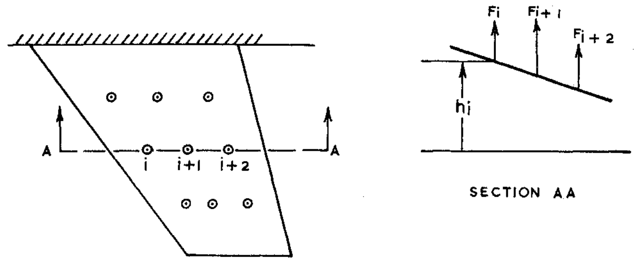


Fig. 1 Wing of general planform.

Outline of the Method

Consider the wing shown in Fig. 1. The control point deflections $\{h\}$ are related to the corresponding forces $\{F\}$ through the flexibility matrix as

$$\{h\} = [a]\{F\} \tag{1}$$

Let the matrix of static aerodynamic influence coefficients $[C_h]$ be defined by

$$\{F\}_{\text{aero}} = q[C_h]\{h\} \tag{2}$$

where $\{F\}_{\text{aero}}$ represents the column matrix of the aerodynamic forces at the control points. (A number of methods are available for computing the $[C_h]$ matrix for wings of arbitrary planform from subsonic to supersonic speeds.)

The value of the dynamic pressure at divergence is obtained when $\{F\}_{\text{aero}}$ in Eq. (2) is the same as the $\{F\}$ in Eq. (1). That is, when

$$[a]^{-1}\{h\} = q_D[C_h]\{h\} \tag{3}$$

or, in the standard form,

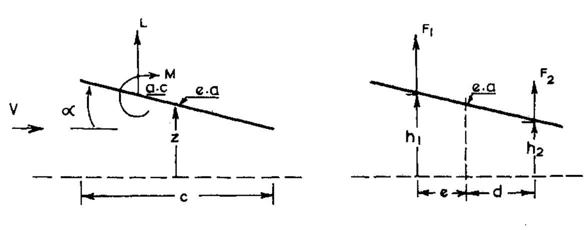
$$(1/q_D)\{h\} = [a][C_h]\{h\} \tag{4}$$

Equation (4) can be solved to obtain the divergence speed. The preceding method is illustrated by application to two subsonic torsional divergence problems in the following section.

Applications

1) The system to be analyzed is shown in Fig. 2a and consists of an aerofoil, restrained at a distance e behind the aerodynamic center by a translational spring of flexibility C^z and a torsional spring of flexibility C^α. For the present analysis, this system is replaced by the equivalent system shown in Fig. 2b. The forward and rear control points are located (arbitrarily) at the aerodynamic centre and at a distance d behind the elastic axis, respectively. For this system, the flexibility matrix is given by

$$[a] = \begin{bmatrix} C^z + e^2C^\alpha & C^z - ed & C^\alpha \\ C^z - edC^\alpha & C^z + d^2 & C^\alpha \end{bmatrix} \tag{5}$$



(a) GIVEN SYSTEM (b) EQUIVALENT SYSTEM

Fig. 2 Two-dimensional model wing.

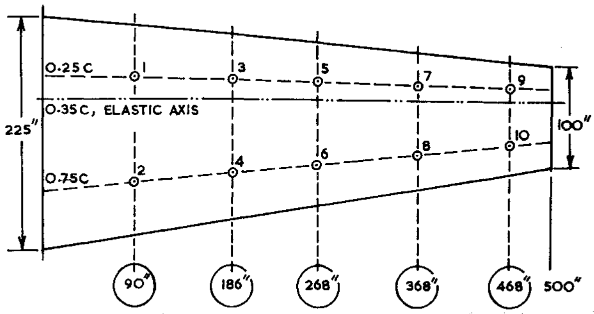


Fig. 3 Jet transport wing.

Using strip theory, it can be shown that the matrix of aerodynamic influence coefficients is given by

$$[C_h] = (S/(e+d)) \begin{bmatrix} 1 & 1/(e+d) \\ 0 & -1/(e+d) \end{bmatrix} \begin{bmatrix} 0 & C_{L\alpha} \\ 0 & c \cdot C_{m\alpha} \end{bmatrix} \begin{bmatrix} d & e \\ 1 & -1 \end{bmatrix} \quad (6)$$

where S is the plan area of the model and c is its chord.

Equations (5) and (6), when combined, give the eigenvalue problem (Eq. 4) which is then solved to obtain the value of q_D. Since the lift and the moment are referred to the aerodynamic centre, $C_{m\alpha} = 0$ and the eigenvalues are

$$(1/q_D) = 0 \qquad S \cdot e \cdot C^{\alpha} \cdot C_{L\alpha} \quad (7)$$

The zero root corresponds to a pure translation mode ($h_1 = h_2$). The second root shows, as expected, that q_D is independent of C^z and is the result obtained from a consideration of moments about the elastic axis.[1]

2) As a second example, the symmetrical divergence speed of a hypothetical jet transport wing, analysed by Bisplinghoff et al. in their book,[1] will be calculated by the present method. The wing is divided into five spanwise stations (Fig. 3) with control points located on the quarter-chord and the three-quarter-chord lines. The flexibility matrix for this system of control points has been calculated by Rodden[2] from data given in Ref. 1. Strip theory aerodynamics was used to set up the matrix of aerodynamic influence coefficients. (The value of $C_{L\alpha}$ was taken as 3.3325/rad.) The results of the analyses using 1, 2, 3, and 5 spanwise stations are shown in Table 1 which also shows the results obtained in Ref. 1 using conventional methods.

One problem encountered in solving Eq. (4) for this particular problem was the following: with the present choice of two control points at each spanwise control station, and with the use of strip theory aerodynamics, Eq. (4) will have as many zero roots as there are spanwise stations. (These correspond to pure bending modes of the wing.) Eq. (4) can be rearranged, in partitioned form, as

$$\left[\begin{array}{c|c} [A] - (1/q_D)[I] & -[A] \\ \hline [B] & -[B] - (1/q_D)[I] \end{array}\right] \left\{\begin{array}{c} h_f \\ \hline h_r \end{array}\right\} = \left\{\begin{array}{c} 0 \\ \hline 0 \end{array}\right\} \quad (8)$$

where the subscript f refers to the control points on the 0.25c line and r refers to those on the 0.75c line. The matrices $[A]$ and $[B]$ are of order $(N \times N)$ where $2N$ is the total number of control points. The zero roots can be eliminated from Eq. (8) and an eigenvalue problem for the nonzero roots can be obtained as

$$(1/q_D)\{h_r\} = [B]([A][B]^{-1} - [I])\{h_r\} \quad (9)$$

The values of $\{h_f\}$ corresponding to a particular eigenvalue can be obtained from

$$[I]\{h_f\} = ([I] + (1/q_D)[B]^{-1})\ \{h_r\} \quad (10)$$

Discussion of the Results and Conclusions

It can be seen from Table 1 that even with three spanwise stations, the divergence speed obtained from the present method is of comparable accuracy to that obtained in Ref. 1 using strip theory. The results of the 5 station analysis using lifting line theory is comparable to the corresponding result of Ref. 1. (Since the aim of these calculations was only to obtain an estimate of the accuracy of the present method, no corrections for compressibility have been applied.) Since no prior assumptions need to be made, this method is applicable to a wing of general planform to predict both the torsional and the chordwise divergence speeds.

References

[1] Bisplinghoff, R. L., Ashley, H., and Halfman, R. L., *Aeroelasticity*, Addison-Wesley Publishing Co., Cambridge, Mass., Chap. 8, 1955, pp. 427–440.

[2] Rodden, W. P., "A Matrix Approach to Flutter Analysis," SMF Fund Paper FF-23, Institute of the Aeronautical Sciences, New York, N.Y. May 1959.

Subsonic Similarity Rule for Jet-Flapped Airfoil

SHELDON ELZWEIG*
Advanced Technology Laboratories Inc., Jericho, N. Y.

Introduction

A SOLUTION of the two dimensional, jet-flapped, symmetric wing in compressible flow by the methods of thin-airfoil theory has been reported in Ref. 1. The analysis assumes that 1) the flow inside the jet is irrotational and bounded by vortex sheets across which it is prevented from mixing with the main stream and 2) the jet is infinitely thin, but possesses finite momentum. It is the aim of this report to extend the analysis, by means of similarity transformations, to the case of a two-dimensional, jet-flapped, symmetric wing in subsonic flow.

Outline of Incompressible Solution

A jet issues from the trailing edge of an airfoil, at angle of attack α_1, and enters an incompressible stream with a deflection τ_1, Fig. 1, where U_1 denotes the freestream velocity, c the chord length, and y_{j_1} the jet shape. In terms of the perturbation velocity potential ϕ the linearized flow equation is

$$\partial^2\phi/\partial x^2 + \partial^2\phi/\partial y^2 = 0 \quad (1)$$

The boundary conditions are mixed. On the wing

$$(\partial\phi/\partial y)_{y=0} = U_1\alpha_1 \qquad 0 < x < c \quad (2)$$

whereas on the jet

$$(\partial\phi/\partial y)_{y=0} = U_1 y_{j_1}'(x) \qquad c < x < \infty \quad (3)$$

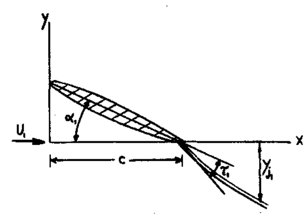


Fig. 1 Jet flap configuration.

Received May 24, 1971; revision received July 7, 1971. The author is grateful to A. Ferri for initiating this work and for his many helpful discussions. Support from the General Dynamics Corporation is gratefully acknowledged.
Index Category: Subsonic and Transonic Flow.
* Research Scientist.